Validation of Approximate Indicial Aerodynamic Functions for Two-Dimensional Subsonic Flow

J. G. Leishman*
University of Maryland, College Park, Maryland

Approximations for two-dimensional indicial (step) aerodynamic responses due to angle of attack and pitch rate are obtained and generalized to account for compressibility effects up to a Mach number of 0.8. Using the Laplace transform method, these indicial functions are manipulated to produce explicit solutions for idealized harmonic forcings. These explicit solutions are subsequently compared with experimentally obtained pitch and plunge aerodynamic data in the reduced frequency domain. The results of this comparison are used to relate back and substantiate the generalization of the compressible indicial lift and moment functions.

Nomenclature

a	= sonic velocity
A_n	= coefficients of indicial functions
b_n	= exponents of indicial functions
C	= airfoil chord
C_M	= pitching moment coefficient about quarter-chord
C_N	= normal force coefficient
$C_{N\alpha}$	= normal force curve slope
k	= reduced frequency ($=\omega C/2U$)
K	= noncirculatory time constant functions
M	= Mach number
q	= normalized pitch rate ($=\dot{\alpha}C/U$)
S	= distance travelled by airfoil in semichords ($=2Ut/C$)
t	= time
T, T'	= noncirculatory time constant in t and S time domains, respectively
T_I	= basic noncirculatory time constant ($=C/a$)
t	= time
x	= chordwise dimension
α	= angle of attack
β	= compressibility factor ($=\sqrt{1-M^2}$)
ϕ	= indicial response function
ω	= circular frequency

Introduction

THE unsteady aerodynamic response of an airfoil to a specific forcing can be computed with some degree of accuracy and detail; however, solutions are generally complex and computational requirements are large. In most rotorcraft applications, which involve many degrees of forcing, the prediction of unsteady airloads and performance render many methods impractical for routine use, and more approximate methods must be devised. Even for the two-dimensional incompressible potential flow case, the calculations are not trivial, but classical solutions have been obtained for the lift and pitching moment on an airfoil under harmonic motion by Theodorsen,[1] Greenberg,[2] and Wagner[3] for a step change between two steady-state conditions. This latter case is termed the indicial response and has the advantage that if the indicial response is known, then the total response to an arbitrary time history of forcing can be obtained using the superposition principle (in the form of Duhamel's integral[4]). Thus, the indicial method offers considerable scope to meet the requirements of rotary wing aeroelastic calculations. However, bearing in mind that rotary wings operate at significant subsonic and transonic Mach numbers, it is clear that allowance for the effect of compressibility must be made in the definition of the indicial force and moment responses for use in these rotor analyses.

Since Wagner's original work on the derivation of the indicial response for incompressible flow, there have been numerous contributions to extend the result to compressible flow, and many of the earlier solutions are reviewed by Bisplinghoff et al.[4] Approximate analytical solutions for the indicial response are quite complicated and are generally only valid for a very brief interval after the initiation of motion. Recently, more elaborate finite-difference solutions for the indicial response have been conducted by Ballhaus[5] using transonic small disturbance equations, and Magnus[6] using Euler equations. These solutions are only feasible at great computational expense and are still subject to computational limitations. Nevertheless, these finite-difference solutions have great potential value in assessing the accuracy and limitations of more approximate theories.

Derivation of the indicial response may also be made using the results of the aerodynamic response due to harmonic motions. Since both the indicial and harmonic responses of Wagner and Theodorsen are solutions to the same linearized form of the partial differential equation governing the flow behavior, they are mathematically equivalent when used to satisfy the same boundary conditions. Küssner[7] appears to have been the first to demonstrate this relationship using an inverse Laplace transform between the Wagner and Theodorsen functions. Garrick[8] also showed the same reciprocal relationship using the direct Laplace transform and Fourier integral methods, where in fact there is a fundamental correspondence between the frequency domain and indicial response through a Fourier transform pair. Since the general relationship between the forcing and the aeodynamic response is the same for a linear aerodynamic theory, the appropriate indicial responses in compressible flow can be obtained in an analogous way. However, the aerodynamic response must be known at a sufficient number of oscillatory frequencies to make a numerical inversion possible.

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*Assistant Professor, Center for Rotorcraft Education and Research, Department of Aerospace Engineering. Member AIAA.

Mazelsky[9] and Mazelsky and Drischler[10] have obtained indicial lift and pitching moment responses for Mach numbers of 0.5, 0.6, and 0.7 using the oscillatory data of Dietze[11] and others, and these results are summarized by Bisplinghoff et al.[4] Recently, a similar approach has been used by Dowell[12] to obtain the indicial response functions at $M=0.7$ and 0.8. General conclusions from these works show that the influence of compressibility on the indicial response manifests itself through a finite and time-dependent initial noncirculatory (or impulsive) behavior, followed by a more gradual approach of the indicial quantities to the final steady values compared with their incompressible counterparts. However, the problem still remains as to how these compressible indicial functions may be best represented and generalized for use in rotorcraft aeroelastic analyses. A common example is the incompressible Wagner function, which can be approximated using a three-term exponential series with four coefficients (two poles).[4] The main advantages of using exponential functions to approximate the indicial response are that they have a simple Laplace transform, and also they facilitate the use of numerical schemes for discretized forcing.[13] Mazelsky[9] and Mazelsky and Drischler[10] have adopted a four-term exponential series approximation to the compressible indicial lift functions. Although their approach appears to be an acceptable procedure for the three Mach numbers presented, they made no attempt to generalize or appropriately scale these functions to other Mach numbers.

Beddoes[13] appears to be the first to note that by appropriately scaling the ordinates of the compressible indicial functions in terms of Mach number alone, they may be reduced to correspond closely to the Wagner function, except for the first few semichord lengths of airfoil travel. Thus, the compressible indicial lift functions may be easily generalized to any subsonic Mach number. In later work, Beddoes[14] extended the generalization of the compressible indicial lift functions to the treatment of the noncirculatory behavior during the first part of the motion. Beddoes postulated that a convenient general representation of the total indicial lift response is to decompose the loading into independent noncirculatory and circulatory parts. The initial value of the noncirculatory loading is given by piston theory (which is a result of acoustics or simple wave theory). This initial loading is subsequently allowed to decay exponentially with time. Simultaneously, the circulatory loading builds up quickly to the steady-state value, and by adjusting the value of the exponent (or time constant) representing the rate of decay of the noncirculatory loading, the behavior of the total indicial response can be maintained.

Heaslet and Spreiter[15] generalized a result given earlier by Sears[16] in which the circulation in a flat plate airfoil plunging indicially is proportional to the lift of the airfoil entering a sharp-edge gust, which for the incompressible case is described by the Küssner function.[7] Using both experimenal and recent finite-difference code results, Beddoes[14] subsequently refined the representation of the circulatory indicial response (similar to the Küssner function) in terms of a two-pole exponential function that scales appropriately with Mach number. Using a supersonic analogy method by which the unsteady subsonic problem is converted to a steady supersonic problem to obtain solutions to the governing linearized two-dimensional differential equation, Lomax et al.[17] have obtained theoretical results for the indicial lift moment behavior due to step changes in angle of attack and pitch rate. The calculations are complex and solutions can be obtained only for a short period of time after the start of the motion (less than half a chord length of airfoil travel), but are sufficient to define the initial behavior of the indicial response. These results have been used by Beddoes[13] to help generalize the total indicial response, and considerable success has been demonstrated when the results are related back to available two-dimensional airfoil test data for harmonic oscillations.

Recent work conducted by Leishman and Beddoes[18] has extended the generalized compressible forms of indicial formulation to the treatment of pitching moment and drag response. The main objective of this work was to incorporate nonlinear aerodynamic effects in a manner consistent with the indicial formulation for use in rotorcraft airloads analysis. The treatment of the noncirculatory loading, however, differs somewhat from the earlier work conducted by Beddoes.[13] Furthermore, the formulation of Ref. 18 does not contain a contribution to the circulatory pitching moment due to pitch rate induced camber effects. Nevertheless, good correlations were demonstrated when the theory was related to airfoil test data in the frequency domain.

The present work has further refined and consolidated the generalized compressible indicial response functions to a higher level of approximation. Revised time constants representing the decay of the noncirculatory lift and pitching moment are derived on a more rigorous basis. Finally, the indicial functions are manipulated to produce explicit solutions for the airfoil lift and moment for harmonic forcing. These solutions are compared with available airfoil test data to validate the indicial functions.

Formulation

By definition, an indicial function is the response to a disturbance that is applied instantaneously at time zero and held constant thereafter, i.e., a disturbance given by a step function. Following Beddoes,[14] two separate components for the total indicial aerodynamic response are used in the present work. First, one is used to represent the initial loading, which is of noncirculatory loading and decays rapidly with time, and second, another is used to represent the circulatory loading, which builds up quickly in the first few lengths of airfoil travel and asymptotes to the appropriate steady-state values. The circulatory component of the indicial response has been shown to be similar to that derived for the penetration of a sharp-edge gust,[16] which for the incompressible case is described by a Küssner function.[7] In Ref. 14, a generalized circulatory lift function was presented for compressible flow. This function is written in terms of a two-pole exponential function that scales with Mach number, the coefficients which represent the response having been selected as a result of both experimental and theoretical analyses. The noncirculatory loading comprises the initial loading on the airfoil due to the instantaneous change in the boundary condition, and is generated by a compression wave on one surface and an expansion wave on the other. The initial loading at $S=0$ can be evaluated from piston theory,[4] which is a result valid for any Mach number. The subsequent decay of the airfoil loading due to the propagation of the initial pressure disturbances can also be represented using exponential functions. The continuity between the initial noncirculatory loading and the succeeding circulatory loading is preserved using superposition.

In general, the indicial lift and quarter-chord pitching moment coefficients for a step change in angle of attack α and a step change in pitch rate q can be approximated by

$$C_{N\alpha}(S) = \left[\frac{4}{M}\phi^I_\alpha(S) + \frac{2\pi}{\beta}\phi^C_\alpha(S)\right] \tag{1}$$

$$C_{M\alpha}(S) = \left[-\frac{1}{M}\phi^I_{\alpha M}(S) - \frac{2\pi}{\beta}\phi^C_\alpha(S)\,[x_{ac}(M) - 0.25]\right]\alpha \tag{2}$$

$$C_{Nq}(S) = \left[-\frac{1}{M}\phi^I_q(S) - \frac{\pi}{\beta}\phi^C_q(S)\right]q \tag{3}$$

$$C_{Mq}(S) = \left[-\frac{7}{12M}\phi^I_{qM}(S) + \frac{-\pi}{8\beta}\phi^C_{qM}(S)\right]q \tag{4}$$

where the indicial functions $\phi_\alpha^C(S)$, $\phi_\alpha^I(S)$, $\phi_{\alpha M}^I(S)$, $\phi_q^I(S)$, $\phi_q^C(S)$, $\phi_{qM}^C(S)$, and $\phi_{qM}^I(S)$ are to be defined. The superscripts C and I refer to the components of circulatory and noncirculatory (impulsive) loading, respectively. The subscript M refers to the pitching moment contribution. Note that the second term in Eq. (2) represents the contribution to the pitching moment due to a Mach number dependent offset of the aerodynamic center from the airfoil quarter-chord. Also, note that the second term of Eq. (4) represents the induced camber pitching moment due to pitch rate motion. The indicial lift function $\phi_\alpha^C(S)$ has been previously defined in Ref. 14 as

$$\phi_\alpha^C(S) = 1 - A_1\exp(-b_1\beta^2 S) - A_2\exp(-b_2\beta^2 S) \tag{5}$$

where $A_1 = 0.3$, $A_2 = 0.7$, $b_1 = 0.14$, and $b_2 = 0.53$. Also, we have $\phi_\alpha^C = \phi_q^C$.

Lomax et al.[17] suggest an interesting analogy with steady supersonic flow from which the indicial responses can be evaluated explicitly between $0 \le S \le 2M/(M+1)$ as

$$C_{N\alpha}(S) = \frac{4}{M}\left[1 - \frac{1-M}{2M}S\right]\alpha \tag{6}$$

$$C_{M\alpha}(S) = -\frac{1}{M}\left[1 - \frac{1-M}{2M}S + \frac{M-2}{4M}S^2\right]\alpha \tag{7}$$

$$C_{Nq}(S) = \frac{1}{M}\left[1 - \frac{1-M}{2M}S + \left(1-\frac{M}{2}\right)\frac{S^2}{2M}\right]q \tag{8}$$

$$C_{Mq}(S) = \frac{1}{M}\left[-\frac{7}{12} + \frac{5(1-M)}{8M}S - \frac{1-M^2}{8M^2}S^2 + \frac{(1-M)^3 + 4M}{64M^2}S^3\right]q \tag{9}$$

If the total response is assumed to consist of an exponentially decaying noncirculatory part and an exponentially increasing circulatory part, as discussed above, then the results of Ref. 17 can be used to estimate the time constants for the noncirculatory decay by matching the gradient of the total response for $S = 0$. This method was originally pursued to a limited extent by Beddoes.[14] For the normal force response to a step change in angle of attack this gives

$$\frac{dC_{N\alpha}(0)}{dS} = \frac{dC_{N\alpha}^I(0)}{dS} + \frac{dC_{N\alpha}^C(0)}{dS} \tag{10}$$

Also, we have

$$C_{N\alpha}^C(S) = \frac{2\pi}{\beta}[1 - A_1\exp(-b_1\beta^2 S) - A_2\exp(-b_2\beta^2 S)] \tag{11}$$

so that

$$\frac{dC_{N\alpha}^C(0)}{dS} = 2\pi\beta(A_1 b_1 + A_2 b_2) \tag{12}$$

For the noncirculatory part we define

$$C_{N\alpha}^I(S) = \frac{4}{M}\phi_\alpha^I(S) = \frac{4}{M}\exp\left(\frac{-S}{T_\alpha'}\right) \tag{13}$$

so that

$$\frac{dC_{N\alpha}^I(0)}{dS} = \frac{-4}{MT_\alpha'} \tag{14}$$

From Eq. (6)

$$\frac{dC_{N\alpha}(0)}{dS} = \frac{-2(1-M)}{M^2} \tag{15}$$

and so equating gradients at $S = 0$ using Eq. (10) and rearranging gives the time constant in the S domain as

$$T_\alpha' = \frac{4M}{2(1-M) + 2\pi\beta M^2(A_1 b_1 + A_2 b_2)} \tag{16}$$

Using the results $T_\alpha = (C/2U)T_\alpha'$ and $M = U/a$ gives the time constant in the t domain as

$$T_\alpha = \left[\frac{2}{2(1-M) + 2\pi\beta M^2(A_1 b_1 + A_2 b_2)}\right]\frac{C}{a} = K_\alpha T_I \tag{17}$$

From Eq. (17) we note that as $M \to 0$ then $T_\alpha \to C/a$, which is a result indicated previously in Ref. 18.

For the indicial pitching moment response due to a step change in angle of attack, a convenient general expression is of the form

$$\phi_{\alpha_M}^I(S) = A_3\exp\left(\frac{-S}{b_3 T_{\alpha_M}'}\right) + A_4\exp\left(\frac{-S}{b_4 T_{\alpha_M}'}\right) \tag{18}$$

Following a similar approach to that above, using Eq. (7) gives the time constant as

$$T_{\alpha_M} = \left[\frac{A_3 b_4 + A_4 b_3}{b_3 b_4(1-M)}\right] = K_{\alpha_M} T_I \tag{19}$$

By comparing the response with Eq. (7) and with Bisplinghoff et al.[4] a good approximation can be made using the values $A_3 = 1.5$, $A_4 = -0.5$, $b_3 = 0.25$, and $b_4 = 0.1$.

For the indicial lift response to a step change in pitch rate about the quarter-chord we have

$$C_{N_q}^C(S) = \frac{\pi}{\beta}[1 - A_1\exp(-b_1\beta^2 S) - A_2\exp(-b_2\beta^2 S)] \tag{20}$$

$$C_{N_q}^I(S) = \frac{1}{M}\phi_q^I(S) = \frac{1}{M}\exp\left(\frac{-S}{T_q'}\right) \tag{21}$$

Using Eq. (8) and following the same procedure as before gives

$$T_q = \left[\frac{1}{(1-M) + 2\pi\beta M^2(A_1 b_1 + A_2 b_2)}\right]\frac{C}{a} = K_q T_I \tag{22}$$

Finally, for the indicial moment response due to a step change in pitch rate about the quarter-chord, the appropriate time constant may be estimated using

$$C_{Mq}^C(S) = \frac{-\pi}{8\beta}\phi_{qM}^C = \frac{-\pi}{8\beta}[1 - \exp(-b_5\beta^2 S)] \tag{23}$$

where $b_5 = 0.5$. For the noncirculatory part we use

$$C_{Mq}^I(S) = \frac{-7}{12M}\phi_{qM}^I = \frac{-7}{12M}\exp\left(\frac{-S}{T_{qM}'}\right) \tag{24}$$

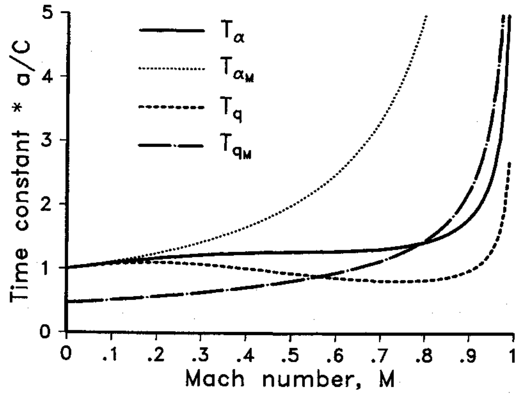

Fig. 1 Variation of noncirculatory time constants with Mach number.

Following the same procedure, using Eqs. (23) and (24) in conjunction with Eq. (9) leads to

$$T_{qM} = \left[\frac{7}{15(1-M)+3\pi\beta M^2 b_5}\right]\frac{C}{a} = K_{qM}T_I \tag{25}$$

The noncirculatory time constants T_α, T_{α_M}, T_q, and T_{qM} are plotted vs Mach number in Fig. 1. Thus, the indicial functions are now defined as functions of Mach number and are plotted in Figs. 2 and 3 to illustrate their behavior. These functions are in excellent overall agreement with those published, for example, in Ref. 4.

Comparison with Finite-Difference Solution

As it is experimentally impossible to validate the indicial response directly, the numerical indicial response calculations performed by Magnus[6] constitute an extremely useful source with which to effect a basic comparison with the present theory. Magnus computed a solution to the Euler equations for an NACA 64A010 airfoil undergoing a step change of incidence of 1 deg at a Mach number of 0.8. The present calculations used the noncirculatory time constants derived above; however, the time constants were slightly adjusted in the light of further correlation studies that are discussed later. Also, the asymptotic lift-curve-slope and an effective steady-state aerodynamic center of 0.315C derived from Ref. 6 were used in the present calculations. The comparison of the computed normal force and pitching moment with the results of Magnus are shown in Fig. 4. The excellent correlation obtained (at considerably less computational effort) provides strong support for the proposed generalization of the indicial lift and moment response functions for higher Mach numbers. The significance of the comparison during the earlier stages of the response (insert), where the noncirculatory terms are important, is of particular interest. It is also significant that, although the Euler calculation was performed for a transonic flow condition, the time history of the lift and moment can be well predicted using the present linear theory.

Aerodynamic Response to Harmonic Forcing

By using the results for the total response due to a prescribed harmonic forcing such as oscillations in pitch or plunge, these data may be used to relate back to the indicial functions. Any discrepancies found may then be used to modify appropriately the indicial functions. For this reason, it is extremely useful to develop explicit solutions for the airfoil lift and moment response due to idealized harmonic motions. As the indicial functions have been completely defined as exponential functions of time, they may be manipulated using the Laplace transform method to produce explicit solutions. For example, the lift and pitching moment response to a har-

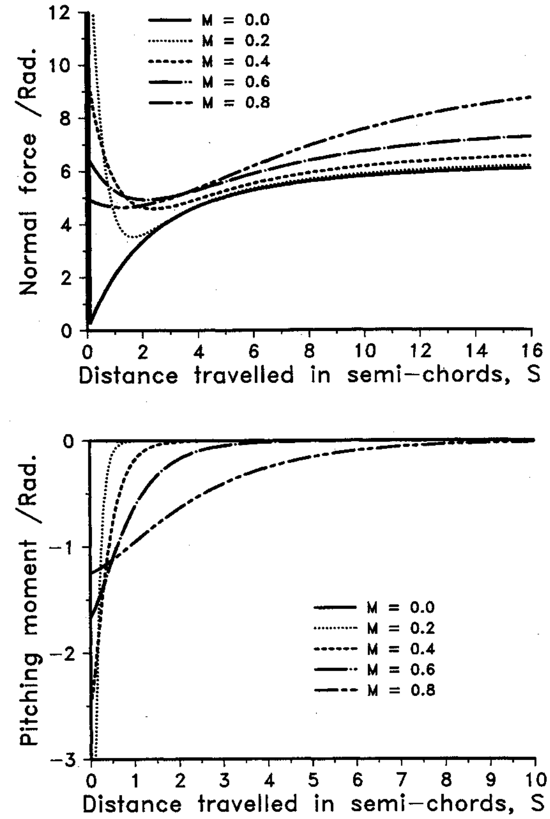

Fig. 2 Indicial normal force and quarter-chord pitching moment response to a step change in angle of attack.

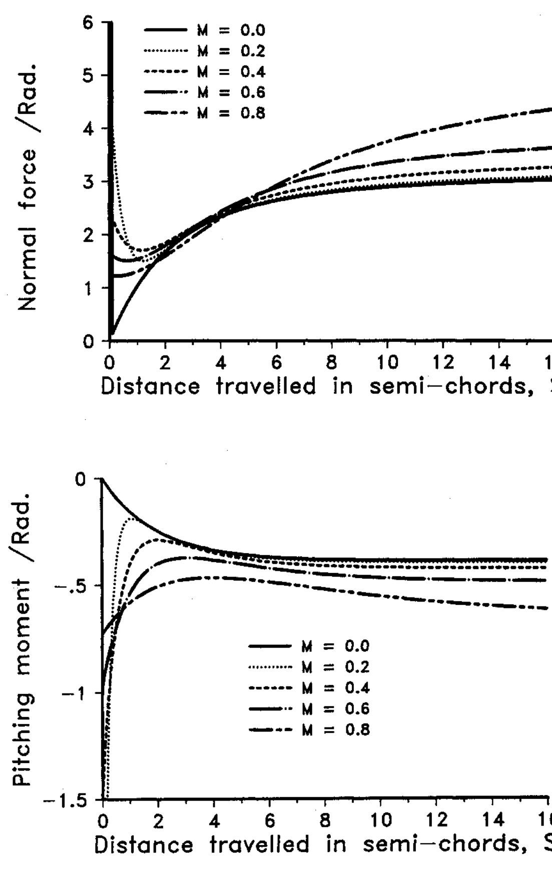

Fig. 3 Indicial normal force and quarter-chord pitching moment response to a step change in pitch rate about the quarter-chord.

monic pitch oscillation about the airfoil quarter-chord axis may be obtained after some manipulaion as

$$\mathrm{Re}C_{N\alpha}(k,M)=C_{N_\alpha}\left[\frac{A_1 b_1^2\beta^4}{b_1^2\beta^4+k^2}+\frac{A_2 b_2^2\beta^4}{b_2^2\beta^4+k^2}\right]+\frac{4}{M}\left[\frac{4K_\alpha^2M^2k^2}{1+4K_\alpha^2M^2k^2}\right] \tag{26}$$

$$\mathrm{Im}C_{N\alpha}(k,M)=-C_{N_\alpha}\left[\frac{A_1 b_1\beta^2 k}{b_1^2\beta^4+k^2}+\frac{A_2 b_2\beta^2 k}{b_2^2\beta^4+k^2}\right]+\frac{4}{M}\left[\frac{2K_\alpha Mk}{1+4K_\alpha M^2k^2}\right] \tag{27}$$

$$\mathrm{Re}C_{M_\alpha}(k,M)=\frac{-1}{M}\left[\frac{4A_3b_3K_{\alpha_M}^2M^2k^2}{1+4b_3^2K_{\alpha_M}^2M^2k^2}+\frac{4A_4b_4K_{\alpha_M}^2M^2k^2}{1+4b_4^2K_{\alpha_M}^2M^2k^2}\right] \tag{28}$$

$$\mathrm{Im}C_{M_\alpha}(k,M)=\frac{-1}{M}\left[\frac{2A_3b_3K_{\alpha_M}Mk}{1+4b_3^2K_{\alpha_M}^2M^2k^2}+\frac{2A_4b_4K_{\alpha_M}Mk}{1+4b_4^2K_{\alpha_M}^2M^2k^2}\right] \tag{29}$$

$$\mathrm{Re}C_{N_q}(k,M)=C_{N_\alpha}\left[\frac{A_1b_1\beta^2k^2}{b_1^2\beta^4+k^2}+\frac{A_2b_2\beta^2k^2}{b_2^2\beta^4+k^2}\right]-\frac{1}{M}\left[\frac{4K_qM^2k^2}{1+4K_q^2M^2k^2}\right] \tag{30}$$

$$\mathrm{Im}C_{N_q}(k,M)=C_{N_\alpha}\left[\frac{A_1\beta^4k}{b_1^2\beta^4+k^2}+\frac{A_2\beta^4k}{b_2^2\beta^4+k^2}\right]+\frac{1}{M}\left[\frac{8K_q^2M^2k^3}{1+4K_q^2M^2k^2}\right] \tag{31}$$

$$\mathrm{Re}C_{M_q}(k,M)=\frac{-C_{N_\alpha}}{8}\left[\frac{b_5\beta^2k^2}{b_5^2\beta^4+k^2}\right]+\frac{7}{12M}\left[\frac{4K_{qM}Mk^2}{1+4K_{qM}^2M^2k^2}\right] \tag{32}$$

$$\mathrm{Im}C_{Mq}(k,M)=\frac{-C_{N_\alpha}}{8}\left[\frac{b_5^2\beta^4k}{b_5^2\beta^4+k^2}\right]-\frac{7}{12M}\left[\frac{8K_{qM}^2M^2k^3}{1+4K_{qM}^2M^2k^2}\right] \tag{33}$$

where Re and Im denote the real and imaginary parts, respectively. The resultant total frequency response can be obtained by summing the various contributions to the real and imaginary components of the lift and pitching moment response. A similar approach can be adopted to find the response to harmonic plunge oscillations which, of course, do not include any q terms. Incremental pitching moments due to a changing aerodynamic center with Mach number can be represented by multiplying the circulatory lift components by the offset of the aerodynamic center from the quarter-chord.

Results and Discussion

Experimental data are available from a number of sources that comprise the unsteady aerodynamic lift and moment response due to pitch and plunge oscillations performed under norminally attached flow conditions, i.e., in the region where linearized aerodynamics are appropriate. It is essential that the data selected be for attached flow conditions, as the presence of aerodynamic nonlinearities due to separation effects introduces further complications in the construction of the indicial responses. (Treatment of nonlinear aerodynamic effects in a form compatible with the indicial method are discussed in Ref. 18.) Both good quality pitch and plunge data are desirable, as the absence of contributing terms due to pitch rate in the plunge oscillations makes it possible to isolate angle-of-attack contributions in the pitch oscillations. However, plunge data is relatively limited in extent and generally seems to exhibit more scatter (particularly in the phase) than is desirable from the point of view of the evaluation of the theory. Pitch oscillation tests are more numerous, and these data provide good scope for evaluation of the theory of a wide range of subsonic and transonic Mach numbers. Starting from a representation of the indicial functions, comparisons of the theoretical and experimental lift and moment frequency response due to pitch and plunge oscillations were made in Refs. 14 and 18. In these works, reasonably good correlations were exhibited with the test data at reduced frequencies appropriate to rotorcraft analysis. The main objective here is to consolidate these calculations and comparisons using the revised analysis.

Oscillatory pitch aerodynamic data were taken from three sources: the Boeing-Vertol facility as documented by Liiva et al.,[19] the Aircraft Research Association (ARA) facility as documented by Wood,[20] and the NASA transonic facility as documented by Davis and Malcolm.[21] Plunge data were taken from both Refs. 19 and 21. The procedure adopted was to compare the present theory with the experimental values of the first harmonic lift and pitching moment amplitude and phase as a function of reduced frequency for the Mach number range of the data.

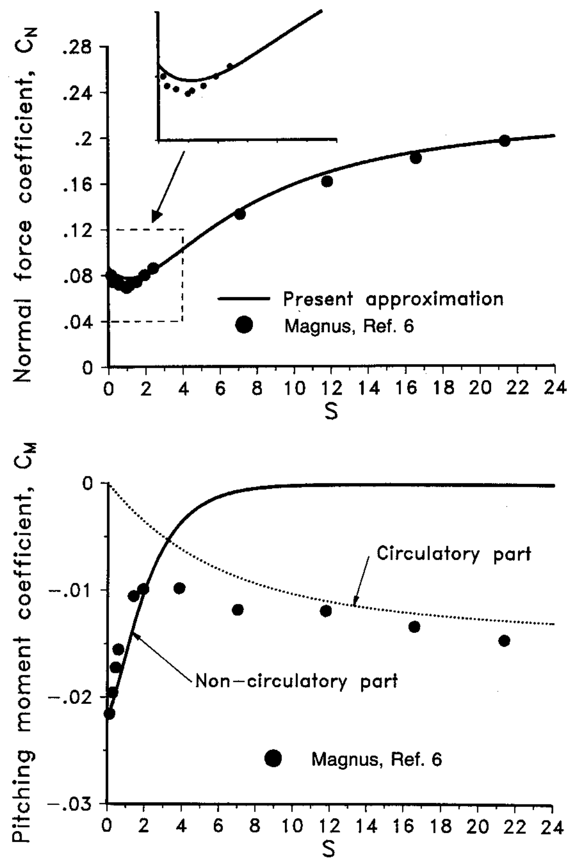

Fig. 4 Comparison of present theory with an Euler code solution for a 1 deg step change in angle of attack at $M=0.8$.

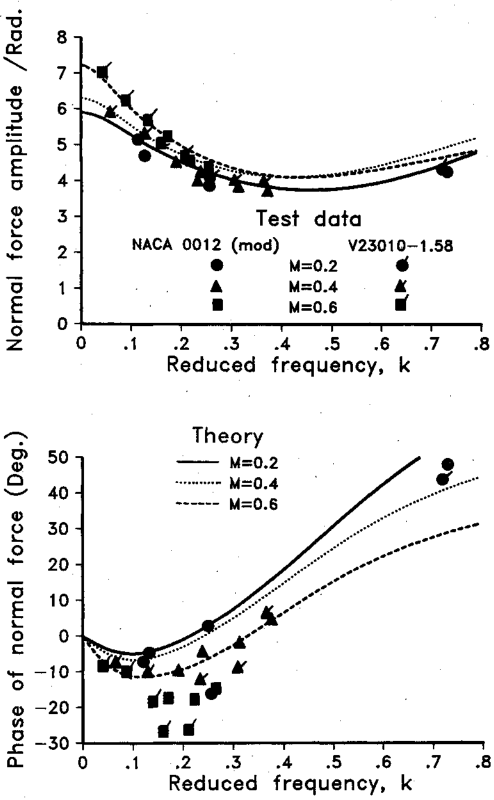

Fig. 5 Comparison of theory with Boeing-Vertol pitch oscillation normal force data.

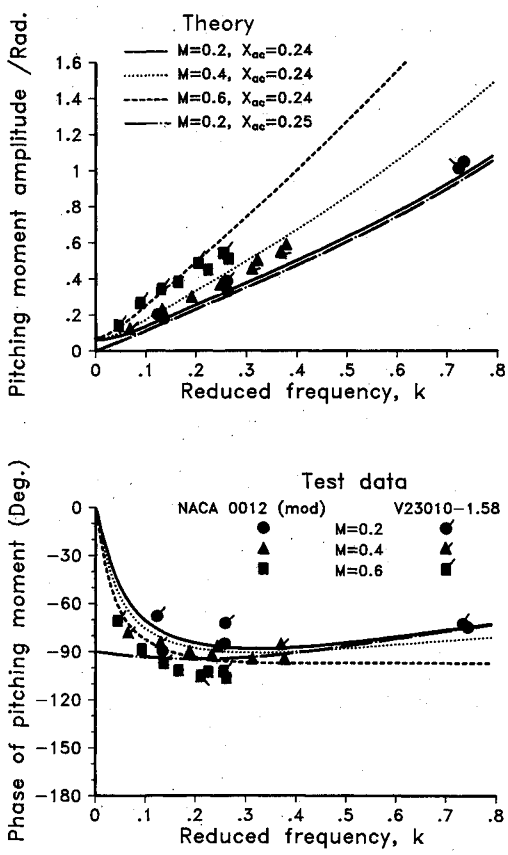

Fig. 6 Comparison of theory with Boeing-Vertol pitch oscillation quarter- chord moment data.

Initial correlation studies led to a number of interesting observations. For both the pitch and plunge oscillation cases, although the theoretical lift amplitude correlated well with the test data, the phase of the response indicated a deviating trend in the higher reduced frequency range. Also, both the amplitude and phase of the pitching moment showed good general qualitative agreement with the test data, but again exhibited a deviation in the higher reduced frequency range. After further consideration, it was determined that a good compromise in terms of optimal correlation with both the pitch and plunge data over the Mach number range was to reduce the time constants T_α and T_q by 25% and the moment time constants $T_{\alpha M}$ and T_{qM} by 20%, respectively. This appeared physically justified because of airfoil thickness, free-edge and viscous effects, which will modify somewhat the initial values and subsequent behavior of the indicial response.

A comparison of the theory with the Boeing-Vertol normal (lift) force and moment data are shown in Figs. 5 and 6, respectively, for Mach numbers of 0.2, 0.4, and 0.6 and for a range of reduced frequency. For the purposes of the comparison, the static lift-curve-slope used in the calculations was matched as closely as possible with the test data. These data are also quite unique, as results at a very high reduced frequency of 0.72 were attained in this experiment, and this provides a good opportunity to extend the range of validity of the theory. Figure 5 shows that there was an excellent correlation with the lift amplitude at all three Mach numbers, and the theory also correlated well with the data taken at $k = 0.72$. Unfortunately, the phase of the lift response did not match as well, especially at $M = 0.6$, even though the qualitative behavior was well represented for the full range of reduced frequency up to 0.72. It should be noted that the change in the sign of the phase (from a lag to a lead) is due to the increasing contribution of the noncirculatory terms as the frequency increases.

Considering the Boeing-Vertol pitching moment data as shown in Fig. 6, as the reduced frequency limited to zero

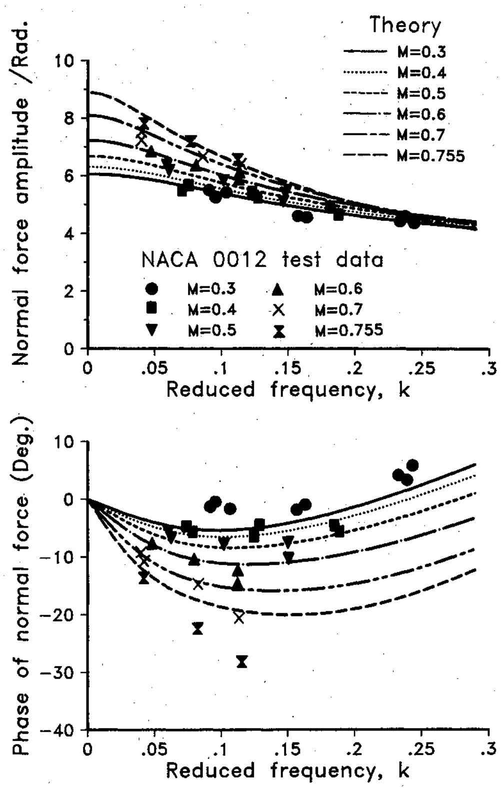

Fig. 7 Comparison of theory with ARA pitch oscillational normal force data for an NACA 0012 airfoil.

(static case) the amplitude of the moment response also limited to near zero indicating that the aerodynamic center (x_{ac}) of the loading on the airfoils was close to the quarter-chord for all three Mach numbers. The behavior of the moment phase response supported this, as there was a fairly good correlation with the theory except at low reduced frequencies. The calculations was subsequently repeated using an offset of the aerodynamic center to 24% chord. In practice this amounted to computing an additional moment contribution from the circulatory component of lift. The results of these revised calculations showed a significant improvement in the phase correlation and was certainly within the range of repeatability and scatter of the data.

Data from the ARA facility for an NACA 0012 airfoil[20] covers a comprehensive range of Mach number from 0.3 to 0.75, and at reduced frequencies up to 0.25. Again, for the purposes of this comparison, the static lift-curve-slopes used in the present theoretical calculations were matched as closely as possible with the test data at the appropriate Mach number. A comparison of the theory with the first harmonic lift amplitude and phase of these test data are shown in Fig. 7. As in the case of the Boeing-Vertol data, the amplitude of the lift response matched well with the test data; however, there was less correlation with the phase of the response at Mach numbers above 0.7 even though the qualitative behavior was well represented.

Quarter-chord pitching moment data for the ARA data are shown in Fig. 8 as a first harmonic amplitude and phase vs reduced frequency. The most noticeable feature of these data was that there appeared to be a significant offset of the aerodynamic center from the quarter-chord axis (cf Boeing-Vertol data). This observation can be concluded mainly from the fact that there was a gradual divergence of the test data from the theory as the reduced frequency tended to zero, even

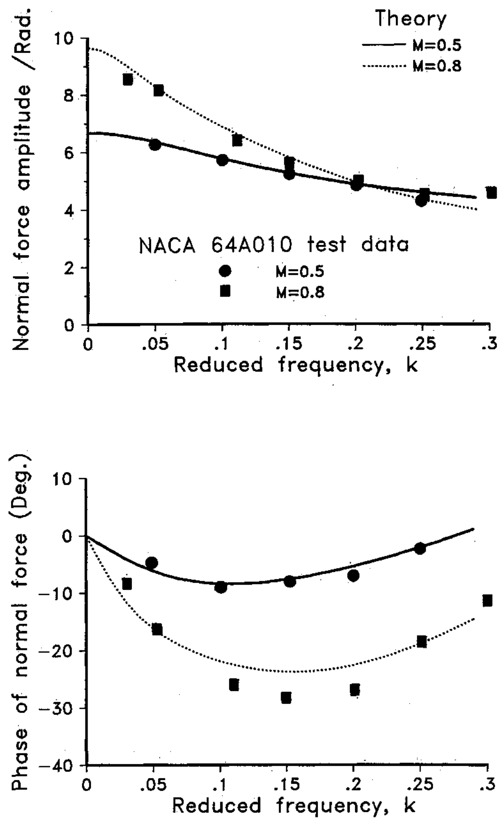

Fig. 9 Comparison of theory with pitch oscillation normal force data for an NACA 64A010 airfoil.

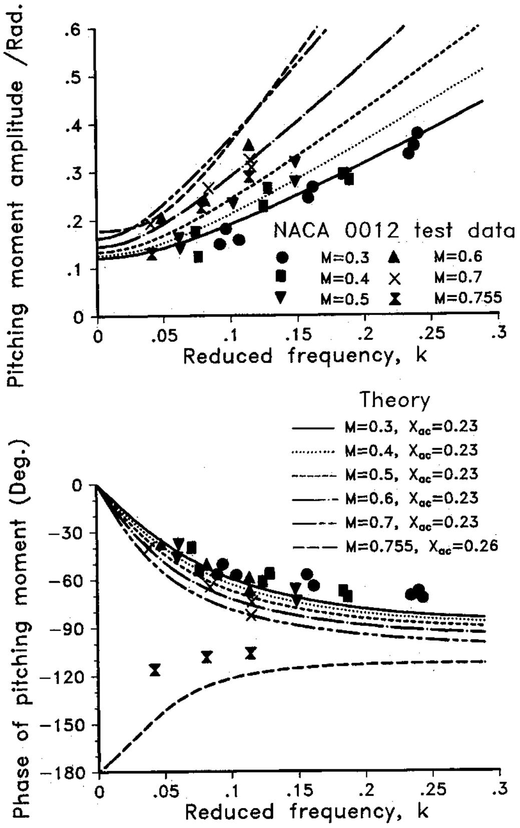

Fig. 8 Comparison of theory using aerodynamic center offset with ARA pitch oscillation quarter-chord moment data for an NACA 0012 airfoil.

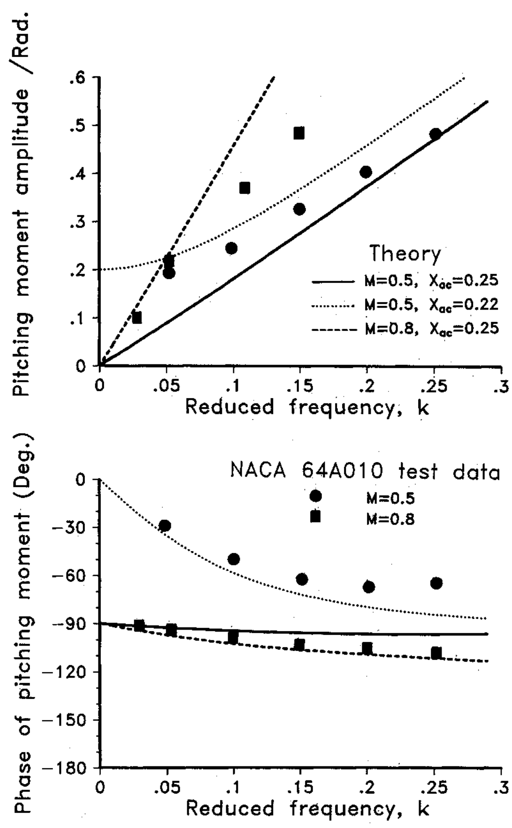

Fig. 10 Comparison of theory with pitch oscillation quarter-chord moment data for an NACA 64A010 airfoil.

though extrapolation of the amplitude data to zero frequency indicated that the aerodynamic center offset was quite small. The present calculations were subsequently repeated using a mean aerodynamic center of 0.23*C* for Mach numbers up to 0.7, and 0.26*C* for a Mach number of 0.755. As indicated in Fig. 8, by offsetting the aerodynamic center good correlations were obtained with the amplitude of the pitching moment response. Furthermore, by including the aerodynamic center offset, the correlation of the theory with the phase of the moment data was significantly improved, and it is worthy to note in Fig. 8 the change in the phase of the response for fore and aft movements of the aerodynamic center from the quarter-chord. It should be emphasized, however, that this form of approach is really only limited to cases where the perturbations are essentially linear about a mean flow. Some of the scatter of the pitching moment phase is also due to the nonlinear effects of trailing-edge separation.

Data obtained by Davis and Malcolm from the NASA facility[21] were for an NACA 64A010 airfoil at Mach numbers of 0.5 and 0.8, and the first harmonic lift and pitching moment response for pitch oscillations are shown in Figs. 9 and 10, respectively. Comparison of the present theory with the lift amplitude and phase of these data was excellent. It is worthy to note for a Mach number of 0.8, a maximum lift phase lag of about 30 deg compared with similar values obtained at a Mach number of 0.6 with some of the Boeing-Vertol data. Considering also the pitching moment response as shown in Fig. 10, a good correlation of theory with the amplitude and phase of the test data was obtained at a Mach number of 0.8. With an appropriate forward movement of the aerodynamic center at a Mach number of 0.5, correlation with these data was also very good, and it should be noted again the significance of the change in the phase of the moment response with a movement of the aerodynamic center.

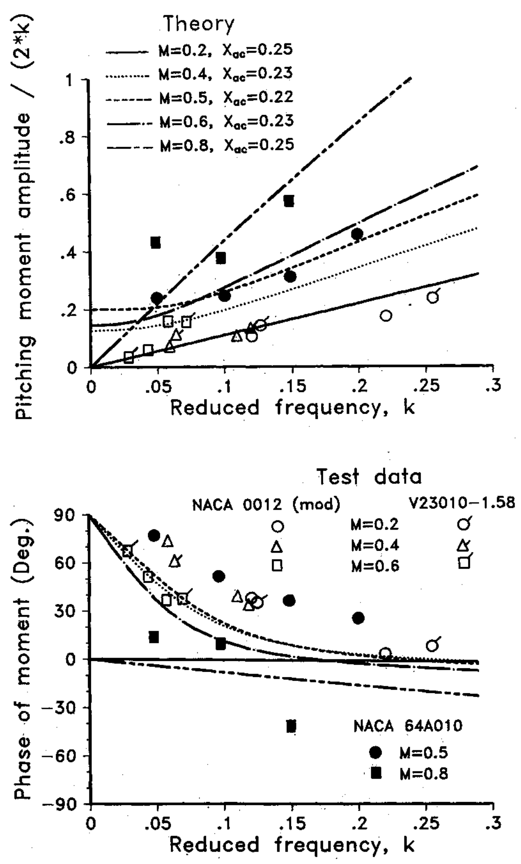

Fig. 11 Comparison of theory with plunge oscillation quarter-chord moment data.

The final set of results presented here are for harmonic plunge oscillations. As noted above, the plunge oscillation data are useful because of the absence of loading contributions due to pitch rate terms. The prediction of the lift response is relatively standard; however, the pitching moment is usually more difficult to predict and is considered here. The theoretical pitching moment is compared in Fig. 11 with the plunge data from Refs. 19 and 21 at Mach nubmers from 0.2 to 0.8 and at reduced frequencies up to 0.3. The moment amplitude correlated well with the test data, this being theoretically proportional to the square of the reduced frequency with the constant of proportionality equal to the noncirculatory time constant T_{α_M}. As noted above, the magnitude of this theoretically derived time constant was reduced by 25% in order to improve the correlation of the theory with the test results. The good correlation obtained from the range of Mach number thus gives credence to the selected value of T_{α_M} and its variation with Mach number. The phase of the moment test data exhibited the characteristic divergence from the theory as the reduced frequency approached zero. As in the case of the pitch oscillations, this characteristic was due to an aerodynamic center offset coupled with some nonlinear trailing-edge separation effects. For the purposes of the present discussion, this effect could be simulated by imposing a mean aerodynamic center offset as detailed above, and a significant improvement in the correlations was found as shown in Fig. 11.

Conclusions

The total indicial lift and pitching moment responses to step changes in angle of attack and pitch rate in subsonic compressible flow have been approximated to engineering accuracy and expressed as general exponential functions of time, which may be scaled in terms of Mach number alone. For generalization purposes, it has been shown how it is convenient to represent the total indicial lift and moment responses as two separate components, one which is of noncirculatory origin and decays rapidly with time, and another which is of circulatory origin and builds up quickly to the appropriate steady-state value. The initial values of the indicial response have been computed using piston theory, and this approach differs from many incompressible analyses where the initial impulse response is determined from apparent mass considerations. Using published analytical solutions to a linearized form of the governing flow equation, it has been shown how the time constants of the approximating indicial functions can be adjusted to maintain the correct total behavior. These time constants also scale with Mach number, and so the appropriate indicial functions may be conveniently represented. The revised indicial functions are currently used within a new version of the aerodynamic model of Ref. 18.

As the aerodynamic indicial response functions have been represented as simple exponential functions of time, they have convenient Laplace transforms. These functions have been manipulated using the Laplace transform method to produce explicit (exact) solutions for harmonic forcing. These explicit solutions are extremely useful, as they allow convenient comparisons with other theoretical and experimental results in the frequency domain independent of any numerical convolution process. A series of comparisons of the computed explicit responses with experimental two-dimensional aerodynamic data for harmonic pitch and plunge oscillations were conducted for a range of Mach numbers up to 0.8 and reduced frequencies up to 0.72. From a general overview of these comparisons in terms of matching the first harmonic lift and pitching moment frequency response for the range of data, a moderate reduction in the values of the theoretically derived noncirculatory time constants was determined to be a satisfactory compromise. This reduction appeared physically justified as finite airfoil thickness and viscous effects will modify somewhat the initial values and subsequent behavior of the noncirculatory loading.

Correlations of the computed lift frequency response with experimental data for pitch and plunge oscillations under nominally attached flow conditions were shown to be satisfactory for a range of Mach number and reduced frequencies normally considered appropriate to rotorcraft analyses. However, some significant discrepancies between the theory and the pitching moment frequency response were noted, especially at low reduced frequencies. By offsetting the aerodynamic center of the circulatory loading from the quarter-chord axis, considerable improvement in the correlation with the measured data was found. This procedure is, however, limited to cases where the unsteady aerodynamic behavior is approximately a linear fluctuation about a mean flow condition. For general application to arbitrary forcing, it is necessary to consider the representation of nonlinear aerodynamic behavior due to flow separation.

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